Naval Research Laboratory

Washington, DC 20375-5000

AD-A240 264

NRL Memorandum Report 6881

Linear Theory of a Quasioptical Gyroklystron With Nonuniform Magnetic Field

R. P. FISCHER, W. M. MANHEIMER, T. A. HARGREAVES AND A. W. FLIFLET

Beam Physics Branch
Plasma Physics Division

September 3, 1991

REPORT DOCUMENTATION PAGE

Form Approved OMB No 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
1991 September 3

3. REPORT TYPE AND DATES COVERED
Interim

4. TITLE AND SUBTITLE
Linear Theory of a Quasioptical Gyroklystron with Nonuniform Magnetic Field

5. FUNDING NUMBERS
JO#47-3786-0-1

6. AUTHOR(S)
R. P. Fischer, W. M. Manheimer, T. A. Hargreaves, and A. W. Fliflet

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Research Laboratory
Washington, DC 20375-5000

8. PERFORMING ORGANIZATION REPORT NUMBER
NRL Memorandum Report 6881

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U. S. Department of Energy
Washington, DC 20545

Office of Naval Research
Arlington, VA 22217

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release
Distribution unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*
A linear analysis of a two-resonator quasioptical gyroklystron is presented which examines the effect of the experimental magnetic field profile and velocity spread of the electron beam. The slow time scale equations of motion are used to determine the effect of a magnetic field taper in the prebunching resonator. Velocity spread is considered by calculating the linear efficiency in the presence of gyrophase mixing in the drift region. The linear efficiency can be made independent of velocity spread to first order by detuning the cyclotron frequencies in the two resonators.

14. SUBJECT TERMS
Quasioptical resonator   Gyroklystron
Prebunching

15. NUMBER OF PAGES
26

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

NSN 7540-01-280-5500

Standard Form 298 (Rev 2-89)
Prescribed by ANSI Std Z39-18
298-102

CONTENTS

LINEAR THEORY OF A QUASIOPTICAL GYROKLYSTRON WITH NONUNIFORM MAGNETIC FIELD

I. Introduction

Gyrotrons are currently under development as high average-power sources of millimeter waves for electron cyclotron resonance heating (ECRH) of fusion plasmas. The leading candidate for ECRH is the waveguide cavity gyrotron, which has achieved powers of 1.2 MW with 20% efficiency at a frequency of 148 GHz in microsecond-length pulses.[1] An alternative configuration is the quasioptical gyrotron (QOG),[2] which has several advantages over the conventional cavity gyrotron as the power and frequency of the radiation are increased.

The resonantor of the QOG is formed by a pair of spherical mirrors separated by many radiation wavelengths. The device can be made to oscillate in the fundamental transverse mode only, since higher-order modes suffer much larger diffraction losses due to the finite size of the mirrors. The axis of the open resonator is oriented perpendicular to the path of the electron beam, so that the radiation is collected separately from the beam. This separation is advantageous for high power operation and facilitates the use of depressed collector technology for higher device efficiency. The peak ohmic heating density on the mirrors, which is important for continuous-wave (cw) devices, is reduced to an allowed level by increasing the separation between the resonator mirrors. Increasing the mirror separation for a given set of mirrors increases the output coupling of the radiation, although the radiation waist at the center of the resonator is insensitive to changes in resonator length. In this manner the output coupling can be varied while the beam-wave interaction length remains approximately constant. This feature is unique to the QOG and provides a technique to optimize the strength of the electric field in the resonator.

The axial mode separation in the QOG is small compared to the interaction bandwidth in cw-relevant devices, therefore multimode effects are important. Most gyrotron applications require

Manuscript approved July 16, 1991.

a single-mode output, so that suppressing unwanted longitudinal modes is of interest in the QOG. Even if multimode operation is permitted, it is not clear whether the optimum efficiency of single-mode operation can be obtained. Multimode theoretical efficiencies are comparable to optimum single-mode efficiencies in the QOG,[3,4] although both of these predictions overestimate experimental results to date.

Recent theoretical work has shown that there exist large regions in operating parameter space where the output is single-moded even though the axial mode density is high.[5] Single-mode operation has been observed up to 250 kW in a cw-relevant QOG at 120 GHz with a 13 μsec pulse length.[6] However, the gyrotron had to be detuned slightly to obtain single-mode operation, which reduced the output efficiency from the multimode value. The output was multimoded at higher powers with as many as 5 or 6 longitudinal modes present at 500 kW output power. The region in parameter space of single-mode operation in the experiement was much less than that predicted by the theory. Theoretical predictions also indicate that tilting the axis of the resonator a few degrees should increase the regions of stable single-mode operation for an annular electron beam. Experimental results indicate that tilting the resonator improves the single-mode efficiency, although the region of single-mode operation does not increase significantly. [6]

One technique which can be used to select the optimum longitudinal mode in the output resonator is to use an external source to set up the rf field at the desired frequency. This may be accomplished by direct injection of radiation into the output resonator[7,8] or by prebunching the electron beam in an upstream resonator.[8] Direct injection requires a high power circulator or isolator so that the external source is not locked by the gyrotron. Such an isolator is difficult to obtain at millimeter wavelengths, so prebunching the electron beam is considered here. A two-resonator quasioptical gyroklystron, shown in Figure 1, comprises a pair of open resonators separated by a field-free drift region. A low amplitude electric field is excited in the first resonator via an external millimeter-wave source, such as an extended interaction oscillator (EIO). Electron cyclotron absorption provides electrons in the prebunching resonator with a "kick" in perpendicular momentum, depending upon their entrance gyrophase, so that the particles gyrate at different frequencies in the drift region. If the strength of the electric field in the first resonator and the length of the drift region are chosen properly, the electrons will arrive at the output resonator strongly bunched in gyrophase angle and will efficiently give up their energy to the wave fields.

In this work we consider the case where the beam current is above the threshhold current of the output resonator in the absence of a prebunching signal. Thus, this regime of operation is fundamentally different than the gyroklystron amplifier. The strength of the prebunching signal and the frequency difference between the two sources has a profound effect on the operation of the gyroklystron. If the prebunching signal is sufficiently large and the frequency separation small, the gyroklystron will operate as a locked oscillator where the equilibrium has a specified phase.[9] Here the steady state operation of the output resonator depends upon the amplitude, frequency, and phase of the prebunching signal. It is in this regime that the electrons are strongly prebunched and the peak gyroklystron transverse efficiency is higher than that of the single cavity gyrotron.[10] The drive signal is present throughout the duration of the oscillation and has a strong effect on the steady state of the oscillator.

A second region of operation is the mode-primed gyroklystron, where the external source is used only during the build-up of fields in the output resonator.[8] The prebunched electron beam selects a particular longitudinal mode in the output resonator, which is given an initial advantage over neighboring modes. If this mode is stable with respect to decay into sidebands, it will grow and nonlinearly suppress satellite modes. The prebunching signal is used to mode-prime the oscillator, and does not affect the frequency of the final state. The main advantage to mode-priming is that the beam premodulation need only be large compared to the noise present at the start of oscillation in the output resonator. This reduces the power requirement for the millimeter-wave source, which is often a limitation at high frequencies.

Another configuration is the mode-locked gyroklystron, where a portion of the output is fed back into the input resonator.[11] The mirror separation of the prebunching resonator is chosen so that only one mode in the output spectrum is resonant in the prebuncher. Thus, the input resonator acts like a filter and the feedback of the desired longitudinal mode is accomplished without an external source. In the mode-locked case the frequency of the oscillator is fixed and there is a relative phase between the two resonators, although the equilibrium doesn't have a selected phase.

An alternative technique which does not require an external source is the gyroklystron oscillator.[12] This device is operated at a beam current slightly above the threshhold current for the prebunching resonator. The electric field in the first resonator will modulate the electron beam and excite that frequency in the output resonator. If the current is increased further, the strong rf fields in the input resonator will introduce a large energy spread to the electrons and spoil the efficient oscillation in the output resonator. Thus, this type of device will only operate over a limited range of parameters.

Two major limitations on conventional cavity gyroklystron experiments are spurious oscillations in the input cavity, output cavity, and drift region and velocity spread of the electron beam. The QOG has a distinct advantage with regard to spurious oscillations because modes other than the TEM_{00q} are strongly discriminated against due to the finite size of the mirrors. Also, only a single longitudinal mode is within the gyrotron interaction bandwidth for small mirror separations. Hence, exciting the proper mode in the input resonator should be straightforward. Velocity spread of the beam is caused predominantly by the electron gun and is difficult to measure and control. Thermal spread manifests itself by debunching the electrons as they drift between the two resonators. The most common means of addressing this problem is to minimize the length of the drift region, which sets a limit on the maximum gyrophase bunching.

The QOG experiment at the Naval Research Laboratory uses a superconducting magnet with a four-inch crossbore where the field is produced by a pair of modified Helmholtz coils. The axial magnetic field has a 7% dip at the center of the crossbore, which is the location of the resonator in previous experiments. For the proposed gyroklystron experiment, the output resonator will remain at the center of the crossbore while the prebunching resonator will be placed upstream, before the magnetic field peak, at approximately the same field as the output resonator. The frequency of the experiment is determined by the availability of a suitable millimeter-wave source, in this case an 85 GHz EIO. The EIO generates 1.5 kW peak power with a pulse length up to 2 μsec and a mechanical tuning range of ± 1 GHz.

This paper is organized as follows. Section II analyzes the effect of a nonuniform magnetic field on the prebunching of the electrons in the first resonator. It is found that a field gradient reduces the electron bunching parameter somewhat but poses no serious limitation to the experiment. Section III examines the effect of velocity spread on the linear efficiency of the gyroklystron. The nonuniform magnetic field can be used to minimize the deleterious effect of thermal spread by detuning the cyclotron frequencies in the two resonators. This calculation determines the position of the prebunching resonator so that the bunching is preserved in the presence of velocity spread in the drift region. Section IV provides a design example where the bunching parameter is calculated for the experimental parameters, while Section V contains conclusions.

II. Linear Theory in the Prebunching Resonator

A. Uniform Magnetic Field

We first consider the case of a uniform magnetic field and calculate the bunching of the electrons due to the electric field in the input resonator and ballistic bunching in the drift region. The equations which describe the spacial evolution of the slow phase and the perpendicular momentum of the particles can be written[3]

$$\frac{d\theta}{dz} = \frac{m}{p_z}(\Omega - \gamma\omega) + \frac{em\gamma}{2p_\perp p_z}E(z)\sin ky_g \sin\theta \tag{1}$$

$$\frac{dp_\perp}{dz} = \frac{-em\gamma}{2p_z}E(z)\sin ky_g \cos\theta. \tag{2}$$

These equations, known as the slow time scale equations of motion, are derived under the assumption that the wave fields evolve on a time scale much longer than the cyclotron period. The beam-wave interaction is taken to occur at the fundamental cyclotron frequency under the weakly relativistic limit ($\gamma - 1 \ll 1$). The slowly varying gyrophase angle is defined $\theta = \psi - \omega t$, where ψ is the particle phase, k is the free space wavenumber, $y_g = y + (p_\perp/m\Omega)\cos\psi$, and the electric field is polarized in the x-direction. The slow phase of an unperturbed particle is

$$\theta = \theta_0 + \frac{m}{p_z}(\Omega - \gamma\omega)z, \tag{3}$$

where $z = 0$ is the position of the prebunching resonator and θ_0 is the random phase of the particle at the entrance of the resonator. In general, the relativistic mass factor (γ), the nonrelativistic cyclotron frequency (Ω), and the parallel momentum of the particle (p_z) are also functions of position.

The mirrors which make up the resonator are chosen to support only the fundamental transverse mode (TEM_{00}). Higher order transverse modes have much larger diffraction losses and typically have quality factors three to five times less than the fundamental mode. The assumed radiation profile is Gaussian

$$E(z) = E_{01}\exp\left[-\frac{z^2}{w_{01}^2}\right]. \tag{4}$$

Here, w_{01} is the radiation waist at the center of a symmetric resonator and the subscript denotes the first resonator. The radiation waist at the center of the resonator is given by[13]

$$w_{01} = \sqrt{\frac{\lambda}{\pi}}\left(\frac{d}{2}\right)^{\frac{1}{4}}(R_c - d/2)^{\frac{1}{4}}. \tag{5}$$

In this equation, d is the mirror separation and R_c is the radius of curvature of the mirrors.

The change in perpendicular momentum for an electron which traverses the prebunching resonator with a low amplitude electric field can be calculated by integrating Eq. (2).

$$\Delta p_{\perp} = \frac{-em\gamma_0}{2p_{z0}} \int_{-\infty}^{\infty} E(z)\sin ky_g \cos[\theta_0 + \frac{m}{p_{z0}}(\Omega - \gamma_0\omega)z]dz. \tag{6}$$

Here it has been assumed that $p_z = p_{z0}$ is a constant of the motion and the relativistic mass factor γ is constant through the first resonator. If the magnetic field is uniform through the prebunching resonator, $\Omega = \Omega_0$ and the integral can be performed analytically yielding

$$\Delta p_{\perp} = -\frac{\sqrt{\pi} em\gamma_0 w_{01}}{2p_{z0}} E_{01}\sin ky_g \cos\theta_0 \exp[\frac{-(\Omega_0 - \gamma_0\omega)^2 w_{01}^2 m^2}{4p_{z0}^2}]. \tag{7}$$

The change in perpendicular momentum is a function of the entrance phase of the particle, so that some electrons gain energy while some lose energy. Note that the change in perpendicular momentum is due to the electric field in the prebuncher and occurs over a distance of approximately $2w_{01}$.

In the field-free drift region,

$$\frac{d\theta}{dz} = \frac{m}{p_{z0}}(\Omega_0 - \gamma\omega) \simeq \frac{m}{p_{z0}}(\Omega - \gamma_0\omega - \frac{p_{\perp}\Delta p_{\perp}}{\gamma_0 m^2 c^2}\omega). \tag{8}$$

Here, the relativistic mass factor has been modified to include the "kick" from the prebunching resonator.

$$\gamma \simeq \gamma_0(1 + \frac{p_{\perp}\Delta p_{\perp}}{m^2c^2\gamma_0^2}) \tag{9}$$

Thus, the slow phase of the particle at the entrance of the second resonator is $\theta = \theta_0 + \Delta\theta$ where

$$\Delta\theta = \frac{m}{p_{z0}}(\Omega_0 - \gamma_0\omega)(L + z) + q\cos\theta_0 \sin ky_g \tag{10}$$

and

$$q = \frac{\sqrt{\pi}\omega p_{\perp} e E_{01} w_{01} L}{2p_{z0}^2 c^2} \exp[-\frac{(\Omega_0 - \gamma_0\omega)^2 w_{01}^2 m^2}{4p_{z0}^2}]. \tag{11}$$

The variable z has been redefined so that $z = 0$ is now at the center of the power resonator and L is the separation between the two resonators. The shape of the electron beam is contained in the $\sin ky_g$ factor, so that pencil beams, annular beams, and sheet beams can be considered. The quantity q is known as the bunching parameter in the gyroklystron literature,[3] and serves as a measure of the phase bunching of the electrons at the entrance of the output resonator. As will be shown later, the linear efficiency is optimized for $q \sim 2$, although somewhat higher values are required in the nonlinear regime.

B. Nonuniform Magnetic Field

In the existing QOG magnet, the field is nonuniform for much of the region preceeding the output resonator. The effect of this nonuniformity on the bunching parameter is calculated in this section.

If the magnetic field is not uniform in the prebunching resonator, the expression for $\Delta p_\perp$ must be modified. The nonrelativistic cyclotron frequency is now a function of position

$$\Omega(z) = \Omega_0 + \delta\Omega(z), \tag{12}$$

where $\delta\Omega(z)$ is a small quantity compared to Ω_0 which contains the nonuniformity of the magnetic field. The change in perpendicular momentum is now

$$\Delta p_\perp = \frac{-em\gamma_0}{2} \int_{-\infty}^{\infty} \frac{E(z)}{p_z(z)} \sin ky_g \cos[\theta_0 + \frac{m}{p_z(z)}(\Omega_0 + \delta\Omega(z) - \gamma_0\omega)z]dz. \tag{13}$$

It is assumed that the energy of the particles is constant with respect to position. This should be a good approximation for the purpose of calculating the momentum change in the first resonator since the electric field is small and the change in particle energy is small ($\Delta\gamma/\gamma_0 \ll 0.01$).

The superconducting magnet currently used in the Naval Research Laboratory's QOG experiment has a magnetic field with a positive taper for much of the region preceeding the power resonator. A plot of the magnetic field versus position is shown in Figure 2. The slope of the magnetic field near the first peak is approximately 0.9 kG/cm at the position $z = -10$ cm. The magnetic field may be written $B = B_0 + \delta B(z)$, where $\delta B(z) = 0.9z$. Here z is given in centimeters, the magnetic field is in units of kG, and $z = 0$ is referenced to the center of the prebunching resonator. This expression may be substituted into Eq. (13) to calculate $\Delta p_\perp$. The parallel momentum of the particle is also a function of axial position due to the variation of the magnetic field. Since $p_\perp^2/B$ is an adiabatic invariant, the change in parallel velocity caused by the nonuniform field is

$$\delta v_z(z) = \frac{-v_{\perp 0}^2}{2v_{z0}} \frac{\delta B(z)}{B_0}. \tag{14}$$

The parallel momentum can now be expressed

$$p_z(z) = p_{z0}[1 - \frac{\alpha^2}{2} \frac{\delta B(z)}{B_0}]. \tag{15}$$

The preceeding expressions for $E(z)$, $p_z(z)$, and $\delta\Omega(z)$ can now be substituted into Eq. 13. Although it is impossible to perform this integration analytically, it is straightforward to implement numerically.

Typical results of the integration of Eq. (13) are shown in Figure 3. The solid curve corresponds to the change in perpendicular momentum for a particle which traverses the prebunching resonator with a uniform magnetic field. The broken curves correspond to magnetic field gradients in the prebunching resonator of 0.5 and 0.9 kG/cm. There is a shift in gyrophase angle between the uniform and nonuniform cases which has been suppressed, since the entrance phase into the prebuncher (θ_0) is arbitrary. The variation in gyrophase shift is smooth in the detuning range between $\pm 5\%$. All of the plots are normalized to the change in perpendicular momentum for the uniform field for zero detuning.

The calculation is performed for a frequency of 85 GHz, a beam voltage of 70 kV, and $\alpha = 1.5$, where $\alpha = v_\perp / v_\parallel$. The mirror separation is 8 cm and the radius of curvature is 20 cm, which correspond to the values of several available mirrors. It is seen that the first resonator must be driven at a frequency close to the relativistic cyclotron frequency for the prebunching to be effective. As the magnetic field gradient is increased, the effectiveness of the bunching decreases. The degradation in bunching is approximately 25% for a magnetic field taper of 0.9 kG/cm for the parameters listed above at zero detuning. The radiation waist for this resonator is 0.94 cm, so that the magnetic field changes approximately $\pm 5\%$ over 4 waist radii. The effect of the nonuniform field can be made smaller by decreasing the radiation waist in the prebunching resonator.

Figure 4 shows the results of the calculation of $\Delta p_\perp$ for $\alpha = 1.0$. Although the shape of the curves is similar, the smaller transverse energy results in a smaller change in perpendicular momentum. Since the $\Delta p_\perp$ scales as $1/p_z$, the ratio between the two curves is approximately 1.27 for a beam voltage of 70 kV. The bandwidth associated with the prebunching is somewhat larger for lower α. This information is important because it is difficult to get an accurate estimate of the electron pitch angle in the experiment. A spread in α in the first resonator will introduce a spread in the bunching parameter, therefore the nonlinear efficiency should be calculated for the resultant distribution in q.

III. Effect of Velocity Spread and Nonuniform Magnetic Field in Drift Region

A. Linear Efficiency in Uniform Magnetic Field

One quantity of interest is the small signal efficiency when a uniform magnetic field is considered. The energy that the particle gives up to the radiation field in the second resonator can be written

$$\Delta W = e\int v_x E_x dt = e\int (p_x/p_z)E_x dz. \tag{16}$$

The expressions for the electric field and the perpendicular momentum are those from linear theory

$$p_x = p_\perp \cos\psi = p_\perp \cos(\omega t + \theta) \tag{17}$$

$$E_x = E_{02}\sin ky_g \exp[-\frac{z^2}{w_{02}^2}]\cos(\omega t + \phi_0). \tag{18}$$

Here, ϕ_0 is the phase difference between the rf fields in the two resonators and the subscript 2 denotes the second resonator. Integrating Eq. (16) and averaging over the gyroperiod yields

$$\begin{aligned}\Delta W(y_g,\theta_0) &= \frac{ew_{02}\sqrt{\pi}E_{02}p_\perp}{2p_{z0}}\exp[-\frac{(\Omega_0-\gamma_0\omega)^2 w_{02}^2 m^2}{4p_{z0}^2}]\sin ky_g\\ &\times\cos[\frac{m(\Omega_0-\gamma_0\omega)}{p_{z0}}L-\phi_0+\theta_0+q\sin ky_g\cos\theta_0].\end{aligned} \tag{19}$$

This expression for ΔW may be integrated over a uniform distribution in entrance phase θ_0 and an arbitrary distribution in the y-direction $f(y_g)$.

$$<\Delta W>= \frac{ew_{02}\sqrt{\pi}E_{02}p_{\perp 0}}{2p_{z0}}\exp[-\frac{(\Omega_0-\gamma_0\omega)^2 w_{02}^2 m^2}{4p_{z0}^2}]\sin[\phi_0-\frac{m(\Omega_0-\gamma_0\omega)}{p_{z0}}L]F(q) \tag{20}$$

where

$$F(q) = \int J_1(q\sin\xi)\sin\xi f_y(\xi)d\xi. \tag{21}$$

The function $F(q)$ depends upon the shape of the beam, where ξ is the variable of integration in the y-direction. For example, a pencil beam gives $F(q) = J_1(q)$, so that the linear efficiency maximizes for $q = 1.84$. It can be seen that the linear efficiency of the gyrolystron is maximum for zero detuning

$$\frac{\Delta\omega}{\omega} = (1-\frac{\Omega_0}{\omega\gamma_0}) = 0. \tag{22}$$

This is in contrast to the single-cavity gyrotron, where the output frequency is always greater than the relativistic cyclotron frequency. However, both the gyrotron and gyroklystron reach optimum efficiency for similar positive detuning values.[10]

B. Velocity Spread and Nonuniform Magnetic Field

A large value of bunching parameter is desired so that the gyroklystron will operate at peak efficiency. This can be accomplished by increasing the strength of the prebunching field and/or increasing the separation between the resonators. A large electric field in the input resonator will give a large perpendicular momentum kick, which will introduce a sizeable spread in energy. An energy spread of several percent is undesireable because the electrons will not interact efficiently with the rf fields in the output resonator.[14] Increasing the separation between resonators makes this region susceptible to velocity spread effects. This section examines how the gyrophase bunching is affected by a distribution in pitch angle and a nonuniform magnetic field.

The change in slow phase in the drift region is

$$\Delta\theta = -\int_0^L (\omega - \frac{\Omega}{\gamma})\frac{dz}{v_z}. \tag{23}$$

If the magnetic field is uniform through the drift region and the electrons are monoenergetic, the change in slow phase can be written

$$\Delta\theta = -(\omega - \frac{\Omega}{\gamma_0})\frac{L}{v_z}. \tag{24}$$

It can be seen that a spread in parallel velocity will cause a distribution in gyrophases for electrons at the second resonator when the detuning between the cyclotron frequency and the bunching frequency is not zero. When this spread in gyrophase is approximately $\frac{\pi}{2}$, the bunching will completely deteriorate in the drift region and the device will operate as a single-cavity QOG.

When the magnetic field in the drift region is not uniform, the cyclotron frequency and parallel momentum can again be written $\Omega(z) = \Omega_0 + \delta\Omega(z)$ and $p_z(z) = p_{z0} - \delta p_z(z)$. It is assumed that $\gamma = \gamma_0$ in the drift region for the present discussion, since only velocity spread of the beam electrons is considered here. The change in slow phase in the drift region is now

$$\Delta\theta = -\int_0^L \frac{dz}{v_{z0}}(\omega - \frac{\Omega_0}{\gamma_0}) + \int_0^L \frac{\delta\Omega(z)}{\gamma_0}\frac{dz}{v_{z0}} + \int_0^L (\omega - \frac{\Omega_0}{\gamma_0})\frac{\delta v_z(z)}{v_{z0}^2}dz. \tag{25}$$

Here the second order term proportional to $\delta v_z(z)\delta\Omega(z)$ has been dropped. The first term of Eq. (25) is a constant with respect to z inside the integral. The other two terms depend upon the nonuniform field. Define the average variation of the magnetic field

$$<\delta B> \equiv \frac{1}{L}\int_0^L \delta B(z)dz. \tag{26}$$

The change in slow phase can now be written

$$\Delta\theta = -\frac{L}{v_{z0}}(\omega - \frac{\Omega_0}{\gamma_0}) + \frac{eL}{m\gamma_0 v_{z0}} <\delta B> + \frac{v_{\perp 0}^2}{2v_{z0}^2}\frac{L}{v_{z0}}(\omega - \frac{\Omega_0}{\gamma_0}) <\delta B> . \tag{27}$$

The technique used in this section is to expand $\Delta\theta$ in terms of the trajectory pitch angle κ, where κ is defined[3]

$$v_{z0} \equiv v_0 \cos\kappa \tag{28}$$

$$v_{\perp 0} \equiv v_0 \sin\kappa. \tag{29}$$

Now,

$$\Delta\theta = \frac{L}{v_0\cos\kappa}\left(\frac{\Omega_0}{\gamma_0}-\omega\right) + \frac{eL}{m\gamma_0}\frac{<\delta B>}{v_0\cos\kappa} - \frac{L<\delta B>}{2v_0\cos\kappa}\left(\frac{\Omega_0}{\gamma_0}-\omega\right)\tan^2\kappa. \tag{30}$$

The change in gyrophase angle can be expanded in terms of $\delta\kappa$ so that

$$\Delta\theta = A_1\delta\kappa + \frac{1}{2}A_2(\delta\kappa)^2 + \Delta\theta_0 \tag{31}$$

where $\Delta\theta_0$ is the change in slow phase for $\kappa = \kappa_0$. The expansion coefficients are

$$\begin{aligned} A_1 &= \frac{L}{v_0}\left(\omega - \frac{\Omega_0}{\gamma_0}\right)\sec\kappa_0\tan\kappa_0 - \frac{eL}{m\gamma_0 v_0} <\delta B> \sec\kappa_0\tan\kappa_0 \\ &\quad + \frac{L}{2v_0}\frac{<\delta B>}{B_0}\left(\omega - \frac{\Omega_0}{\gamma_0}\right)[2+3\tan^2\kappa_0]\sec\kappa_0\tan\kappa_0 \end{aligned} \tag{32}$$

$$\begin{aligned} A_2 &= \frac{L}{v_0}\left(\omega - \frac{\Omega_0}{\gamma_0}\right)\sec\kappa_0[1+2\tan^2\kappa_0] - \frac{eL}{m\gamma_0 v_0} <\delta B> \sec\kappa_0[1+2\tan^2\kappa_0] \\ &\quad + \frac{L}{2v_0}\frac{<\delta B>}{B_0}\left(\omega - \frac{\Omega_0}{\gamma_0}\right)\sec\kappa_0[2+13\tan^2\kappa_0+12\tan^4\kappa_0]. \end{aligned} \tag{33}$$

The linear efficiency of the gyroklystron is a convenient measure of the effect of velocity spread that can be treated analytically. Thermal spread of electron velocities will principally effect the sine factor of Eq. (20) because its argument is integrated over the entire drift region. The exponential factor is due to the force bunching in the output resonator where the integration is performed over $2w_{02}$. Thus, the linearized efficiency with velocity spread can be calculated by averaging over the sine factor if the drift region is much longer than the radiation waist ($L \gg w_{01}$, w_{02}). Assume a Gaussian distribution in pitch angle $\kappa = \kappa_0 + \delta\kappa$

$$f(\delta\kappa) = \frac{1}{\sqrt{2\pi}a}\exp\left[-\frac{\delta\kappa^2}{2a^2}\right]. \tag{34}$$

The expression for $< \Delta W >$ may now be averaged over the distribution in κ.

$$\begin{aligned} < \Delta W > &= \frac{1}{2}ew_{02}\sqrt{\pi}E_{02}\tan\alpha_0\exp\left[-\left(\omega-\frac{\Omega_0}{\gamma_0}\right)^2 w_{02}^2/4v_{z0}^2\right]F(q)\sin(\phi_0-\overline{\theta}) \\ &\quad \times(1+a^4A_2^2)^{-1/4}\exp\left[-\frac{1}{2}A_1^2a^2/(1+a^4A_2^2)\right]. \end{aligned} \tag{35}$$

In the preceeding equation,

$$\bar{\theta} = \Delta\theta(\kappa_0) + \frac{1}{2}\tan^{-1} A_2 a^2 - \frac{1}{2} A_1^2 A_2 a^4 / (1 + a^4 A_2^2). \tag{36}$$

The small signal efficiency of the gyroklystron without thermal spread is obtained by setting $a = 0$ in the preceeding equations. When thermal spread is considered, the coefficients A_1 and A_2 modify the expression for efficiency in Eq. (35). These coefficients depend upon the separation between resonators L, the mean value of α, and the magnetic field nonuniformity $< \delta B >$. Define the velocity spread coefficient

$$C(a) = (1 + a^4 A_2^2)^{-1/4} \exp[-\frac{1}{2} A_1^2 a^2 / (1 + a^4 A_2^2)]. \tag{37}$$

This coefficient describes the degradation of the linear efficiency due to velocity spread and varies between 0 and 1. The result of the calculation is shown in Figure 5 for an average nonuniformity of the magnetic field of 4.1%, which is the value for the QOG magnet for $L \sim 10$cm. The mean value of pitch angle is $\kappa_0 = 1.0$ ($\alpha = 1.56$), while the three curves correspond to pitch angle spreads $a = 0.025$, 0.05, and 0.1. The corresponding values of $\Delta\alpha$ are approximately 5%, 10%, and 20%. The velocity spread coefficient maximizes at a frequency detuning of approximately 3.6%, which is the difference in cyclotron frequencies in the two resonators.

Figure 6 shows the results of the calculation of the thermal coefficient for $\kappa_0 = 0.8$, which corresponds to $\alpha = 1.0$. The full width at half maximum for the three curves is a =0.025, 0.05, and 0.10, yielding similar values for $\Delta\alpha$. The maximum of the curves is shifted slightly to the right and now occurs at $\Delta\omega/\omega = 3.7\%$. The point where the effect of spread is minimized depends only weakly upon the average value of α, so that fixing the frequency detuning doesn't constrain the allowed values of $v_\perp/v_\parallel$. The main dependence of the optimum frequency detuning is on the average nonuniformity of the magnetic field. The magnetic field shape can be varied somewhat in the experiment, which allows for a technique to tune the optimum frequency difference between the input and output resonators. Another feature of Figure 6 is that the bandwidth of the interaction increases with decreasing α_0 for large velocity spreads. Thus, it may be desireable to operate in this regime to lower the effect of thermal spread.

IV. Estimation of the Bunching Parameter

The condition for starting oscillation in the first resonator with an annular electron beam with optimum frequency detuning can be expressed[2,4]

$$P_b Q \geq \frac{2c^5 \epsilon_0 m^2}{e^2} \exp\left[\frac{1}{2}\right]\left(\frac{d}{w_{01}}\right)\gamma_0(\gamma_0 - 1)\frac{\beta_{z0}^3}{\beta_{\perp 0}^2}\frac{2}{1 \pm J_0(kr_b)}. \tag{38}$$

In the above equation P_b is the input beam power in Watts, d is the mirror separation, r_b is the beam radius, and β is the ratio of the particle velocity to the speed of light. Considering the annular beam is important because some electrons pass through nulls of the standing fields in the resonator, which decreases the efficiency of the interaction. If the guiding center radius r_b is somewhat greater than the wavelength, then $J_0(kr_b) \sim 0$ and Eq. (38) can be written

$$P_b Q \geq 4.6 \times 10^9 \left(\frac{d}{r_{01}}\right)\gamma_0(\gamma_0 - 1)\frac{\beta_{z0}^3}{\beta_{\perp 0}^2}. \tag{39}$$

As an example, consider a resonator with a mirror separation of 8 cm and mirror radius of curvature 20 cm. For a frequency of 85 GHz, the radiation waist is 0.94 cm. An electron beam voltage of 70 kV corresponds to a relativistic mass factor $\gamma_0 = 1.137$, and assuming $\alpha = 1.5$ yields $\beta_{\perp 0} = 0.40$ and $\beta_{z0} = 0.266$. If the quality factor (Q) of the resonator is 1000, then the threshhold current of the prebunching resonator is 9.6 A. The Q of the input resonator is determined by the diameter of the mirrors and the size of the coupling hole.

The electric field in the bunching resonator determines the magnitude of the bunching parameter. A fundamental relation between the energy stored in the resonator and power dissipated by losses is

$$Q = \omega W_{em}/P_l, \tag{40}$$

where W_{em} is the stored electromagnetic energy and P_l is the average power lost. For a Fabry-Perot resonator, the stored energy can be written[15]

$$W_{em} = \frac{\pi}{8}\epsilon_0 w_{01}^2 d E_0^2. \tag{41}$$

Here, ϵ_0 is the permeability of free space and all quantities are in MKS units. For the resonator described above and an input power of 375 Watts, the electric field at the center of the resonator is $E_{01} = 1.67 \times 10^5$ V/m. The total power available from the EIO is 1500 Watts, so the value 375 Watts should be a conservative estimate of the power coupled into the TEM_{00} mode.

If the drive frequency is sufficiently close to the relativistic cyclotron frequency in the first resonator, then the exponential term in Eq. (11) is approximately one. When the effect of the magnetic field taper is included, the bunching parameter can be written

$$q \sim (0.75)\sqrt{\pi}\omega p_\perp e E_{01} w_{01} L / 2p_{z0}^2 c^2. \tag{42}$$

The separation L between the two resonators is 10 cm for a detuning of 3.6%, yielding a bunching parameter $q = 1.8$. Tran *et al.* have shown that the optimum bunching parameter for peak efficiency with a pencil beam placed on a field maximum is approximately 3.[10] The optimum value of q for an annular beam is somewhat greater due to variation of electric field across the beam. Thus, the maximum bunching in this example is somewhat less than optimum.

It can be seen from Eq. (11) that the bunching parameter scales as $p_\perp/p_z^2$, so that reducing $v_\perp/v_\parallel$ to 1.0 reduces the bunching parameter by approximately 50% for a beam voltage of 70 kV. This operating regime (low α) is of interest because velocity spread effects will be smaller and the gyroklystron can operate at higher current before the prebunching resonator oscillates

V. Conclusions

The slow time scale equations of motion have been linearized to study a quasioptical gyroklystron with a nonuniform magnetic field in the prebunching resonator and drift region. The prebunching resonator must be driven close to relativistic cyclotron frequency for the prebunching to be effective. A tapered field in the input resonator is shown to reduce the bunching parameter by approximately 25% for a taper of $\pm5\%$ over 4 waist radii. This effect can be reduced by decreasing the radiation waist in the prebunching resonator.

The equations of motion are integrated through the drift region to examine the effect of velocity spread coupled with the nonuniform magnetic field. It is assumed that the drift region is much longer than the radiation waist in the two resonators so that the predominant effect of velocity spread is phase mixing of the beam. For the experimental parameters chosen here, a frequency detuning of 3.6% minimizes the effect of the velocity spread for the magnetic field variation of 4.1%. Thus, the difference between the cyclotron frequencies in the two resonators should be fixed at 3.6%, which should result in good nonlinear efficiency. For the available millimeter-wave source, a maximum bunching parameter of $q \sim 1.8$ is obtained. Varying the bunching from small to large values should allow gyroklystron operation in several different regimes so that mode selection and efficiency enhancement can be investigated.

Acknowledgements

This work was supported by the Office of Fusion Energy of the U.S. Department of Energy and by the Office of Naval Research.

References

[1] K. Kreischer, private communication.

[2] P. Sprangle, J.L. Vomvoridis, and W.M. Manheimer, "Theory of the quasioptical electron cyclotron maser," *Phys. Rev. A*, vol 23, no. 6, pp. 3127-3138, 1981.

[3] A. Bondeson, W.M. Manheimer, and E. Ott, "Multimode Analysis of Quasioptical Gyrotrons and Gyroklystrons," Infrared and Millimeter Waves Vol. 9, Academic Press, N.Y., N.Y., pp. 309-340, 1983.

[4] M.Q. Tran, A. Bondeson, A. Perenoud, S. Alberti, B. Isaak, and P. Muggli, "Multimode simulation of the frequency spectrum of a quasioptical gyrotron," *Int. J. Electronics*, vol. 61, no. 6, pp. 1029-1040, 1986.

[5] T.M. Antonsen, Jr., B. Levush, and W.M. Manheimer, "Stable single mode operation of a quasioptical gyrotron," *Phys. Fluids B*, vol. 2, pp419-426, 1990.

[6] T.A. Hargreaves, A.W. Fliflet, R.P. Fischer, M.L. Barsanti, W.M. Manheimer, B. Levush, and T.M. Antonsen, Jr., "Tilted resonator experiments on a quasioptical gyrotron," see this issue.

[7] M. Read, R. Seeley, and W. Manheimer, "Observation of phase locking in a single cavity gyrotron oscillator," *IEEE Trans. Plasma Sci.*, vol. PS-13, pp. 398-404, 1985.

[8] A.H. McCurdy and C.M. Armstrong, "Mode selection by priming in an overmoded electron cyclotron maser," *Phys. Fluids B*, vol. 3, no. 1, pp. 212-227, 1991.

[9] A.H. McCurdy, C.M. Armstrong, W.M. Bollen, R.K. Parker, and V.L. Granatstein, "Improved Oscillator Phase Locking by Use of a Modulated Electron Beam in a Gyrotron," *Phys. Rev. Letters*, vol. 57, no. 19, pp. 2379-2382, November 10, 1986.

[10] T.M. Tran, B.G. Danly, K.E. Kreischer, J.B. Schutkeker, and R.J. Temkin, "Optimization of Gyroklystron Efficiency," *Phys. Fluids*, vol. 29, pp. 1274-1282, 1986.

[11] W.M. Manheimer, B. Levush, and T.M. Antonsen, Jr., "Equilibrium and Stability of free-running, phase-locked, and mode-locked quasioptical gyrotrons," *IEEE Trans. Plasma Sci.*, vol. 18, no. 3, pp. 350-368, 1990.

[12] A.K. Ganguly and K.R. Chu, "Analysis of two-cavity gyroklystron," *Int. J. Electronics*, vol. 51, no. 4, pp. 503-520, 1981.

[13] A. Yariv, *Introduction to optical electronics*, Holt, Rinehart, and Winston, N.Y., p. 69, 1976.

[14] G. Saraph, G.I. Lin, T.M. Antonsen, Jr., and B. Levush, "Regions of single mode operation in a high power gyrotron oscillator," in *Fifteenth International Conference of Infrared and Millimeter Waves,* (R.J. Temkin, ed.), p. 511, SPIE, Bellingham, WA, 1990.

[15] T.A. Hargreaves, R.P. Fischer, R.B. McCowan, and A.W. Fliflet, "Ohmic effects in quasioptical resonators," *Int. J. Infrared Millimeter Waves,*" vol. 12, no. 1, pp. 9-22, 1991.

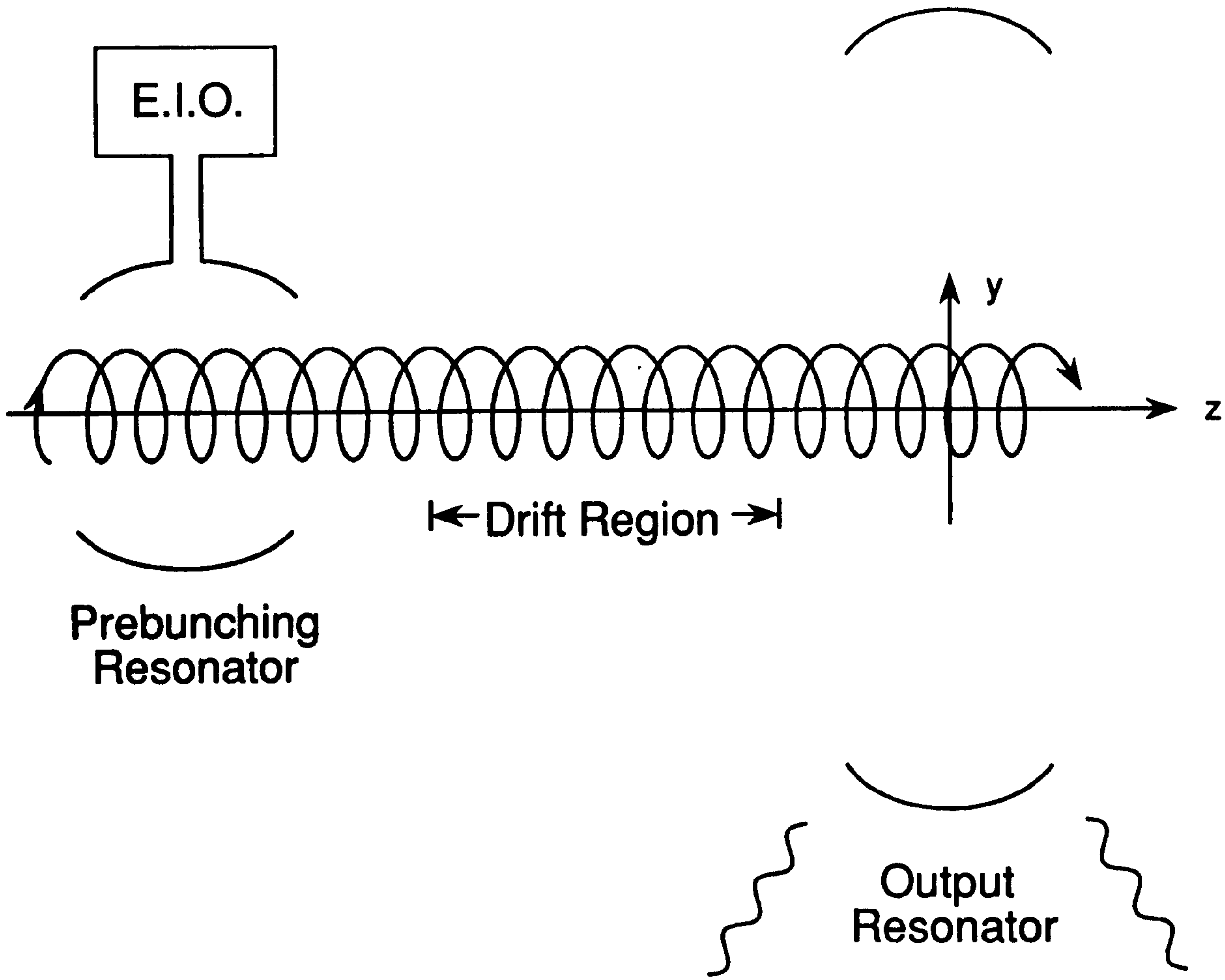

Fig. 1 — Schematic diagram of a two-resonator quasioptical gyroklystron

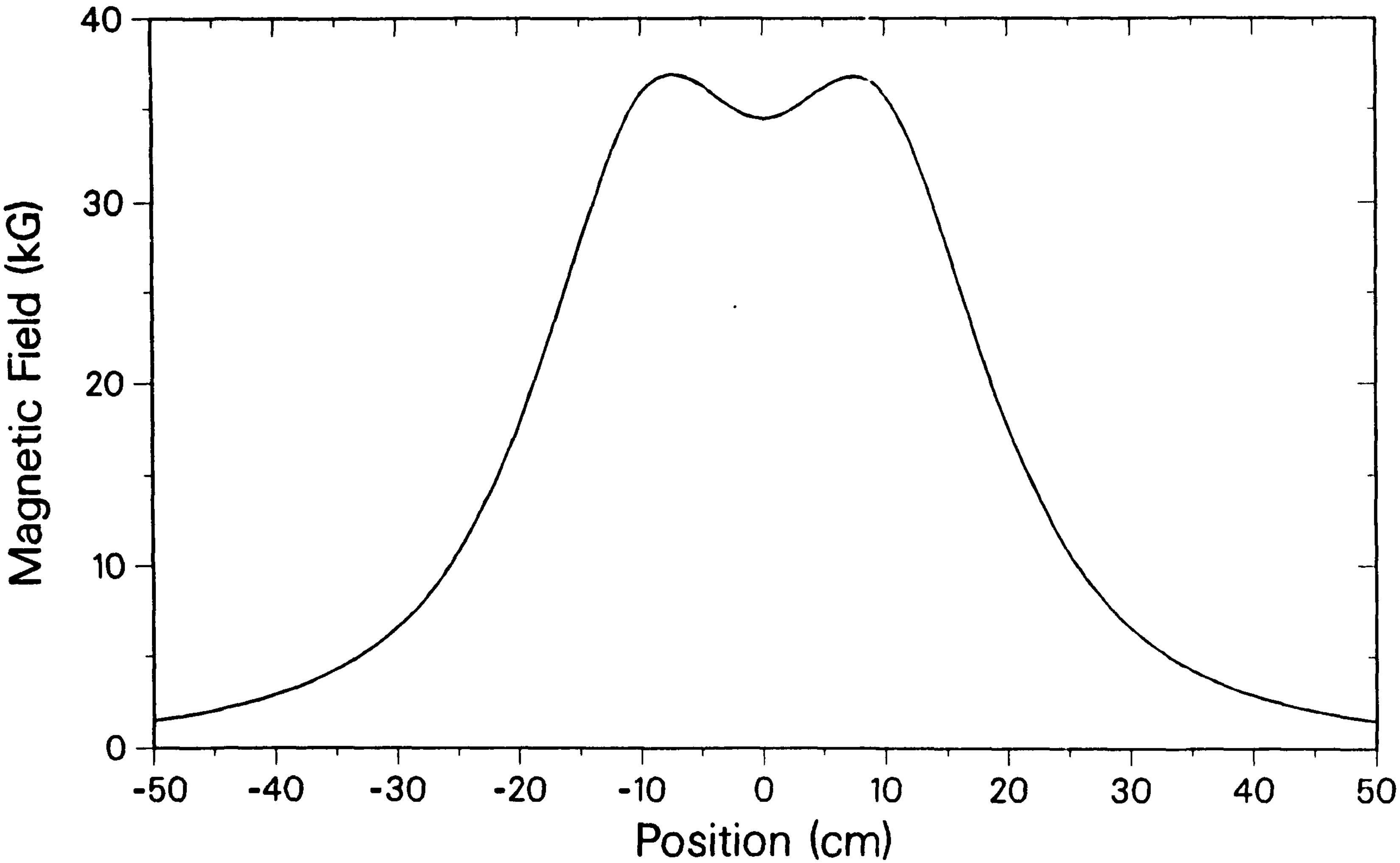

Fig. 2 — Magnetic field profile of the superconducting magnet used in the QOG experiments

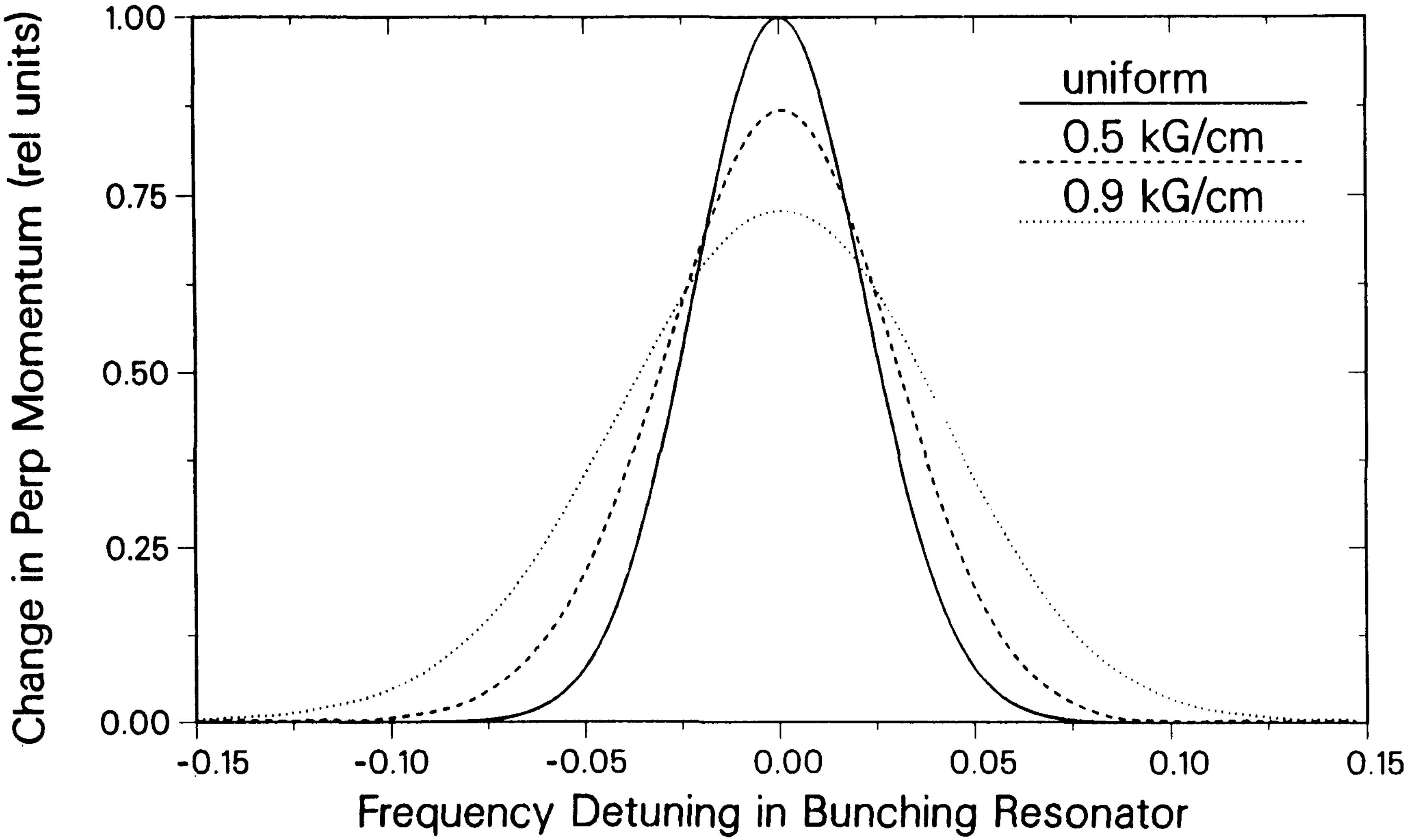

Fig. 3 — Normalized bunching parameter versus frequency detuning in the prebunching resonator for $\alpha = 1.56$. The solid, dashed, and dotted curves correspond to magnetic field gradients of 0.0, 0.5, and 0.9 kG/cm, respectively.

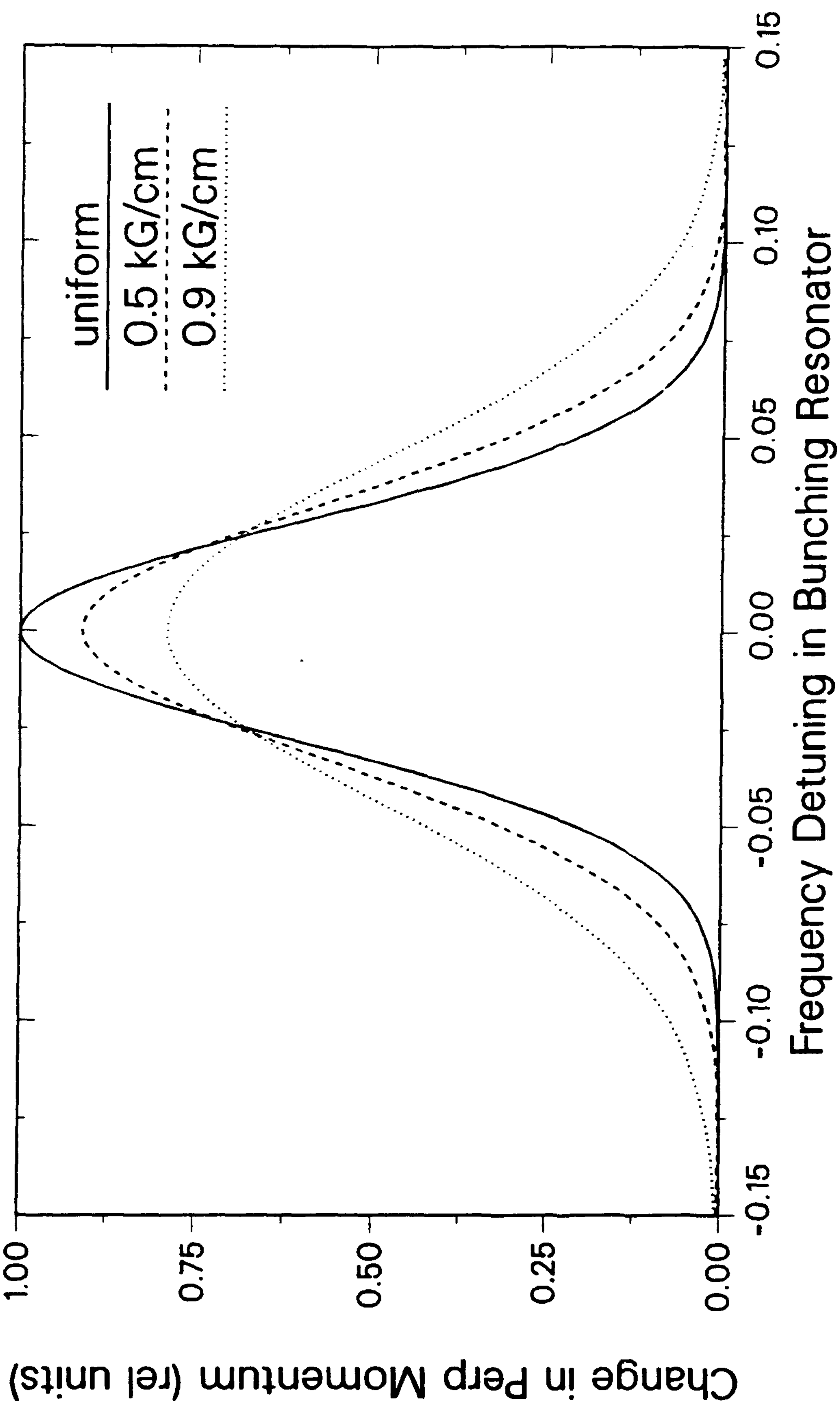

Fig. 4 — Same as Figure 3 for α = 1.0

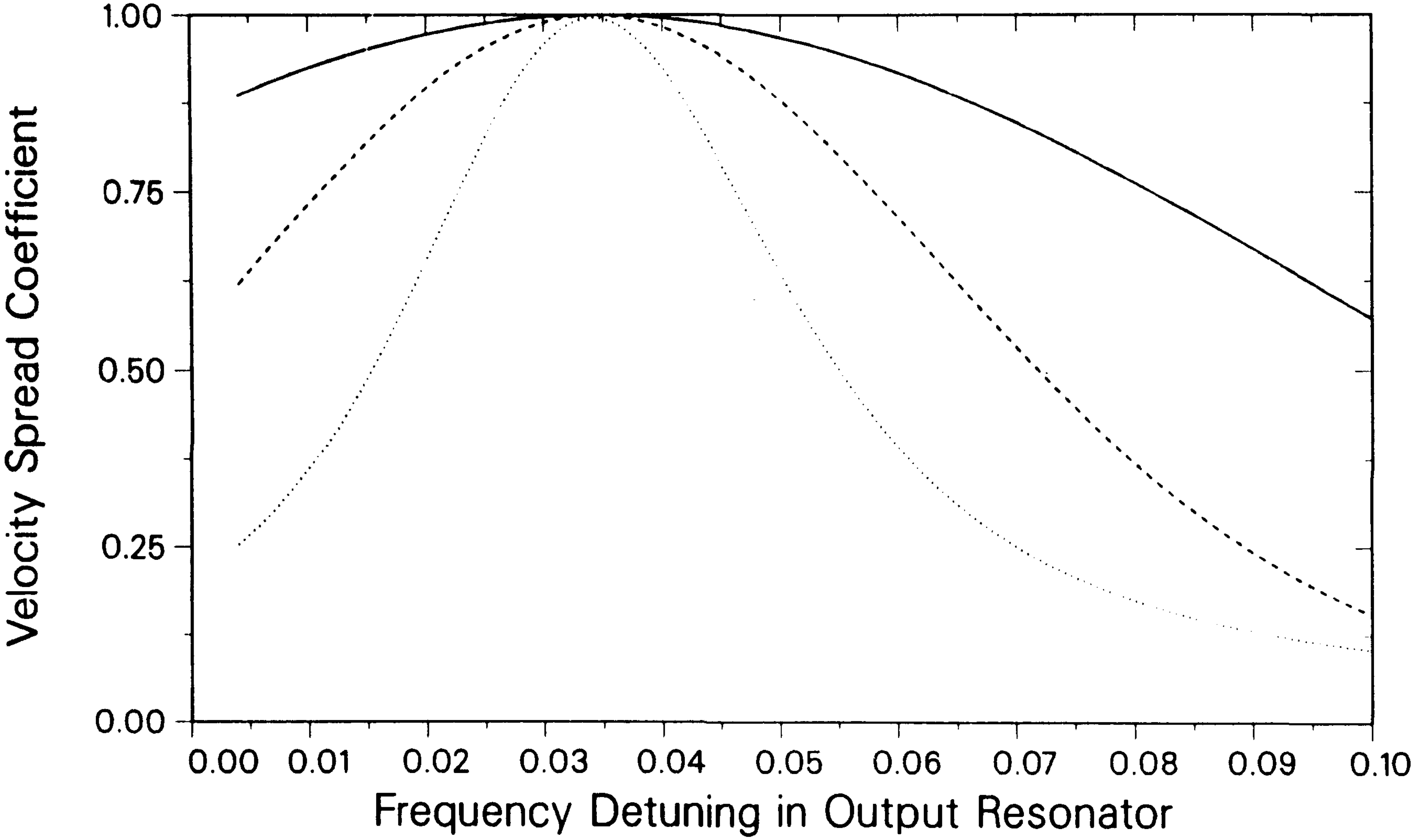

Fig. 5 — Velocity spread coefficient, defined in Eq. (37), vs frequency detuning. The pitch angle distribution is Gaussian with mean $\varkappa_0 = 1.0$ and standard deviation $a = 0.025, 0.05$, and 0.1 for the solid, dashed, and dotted curves, respectively.

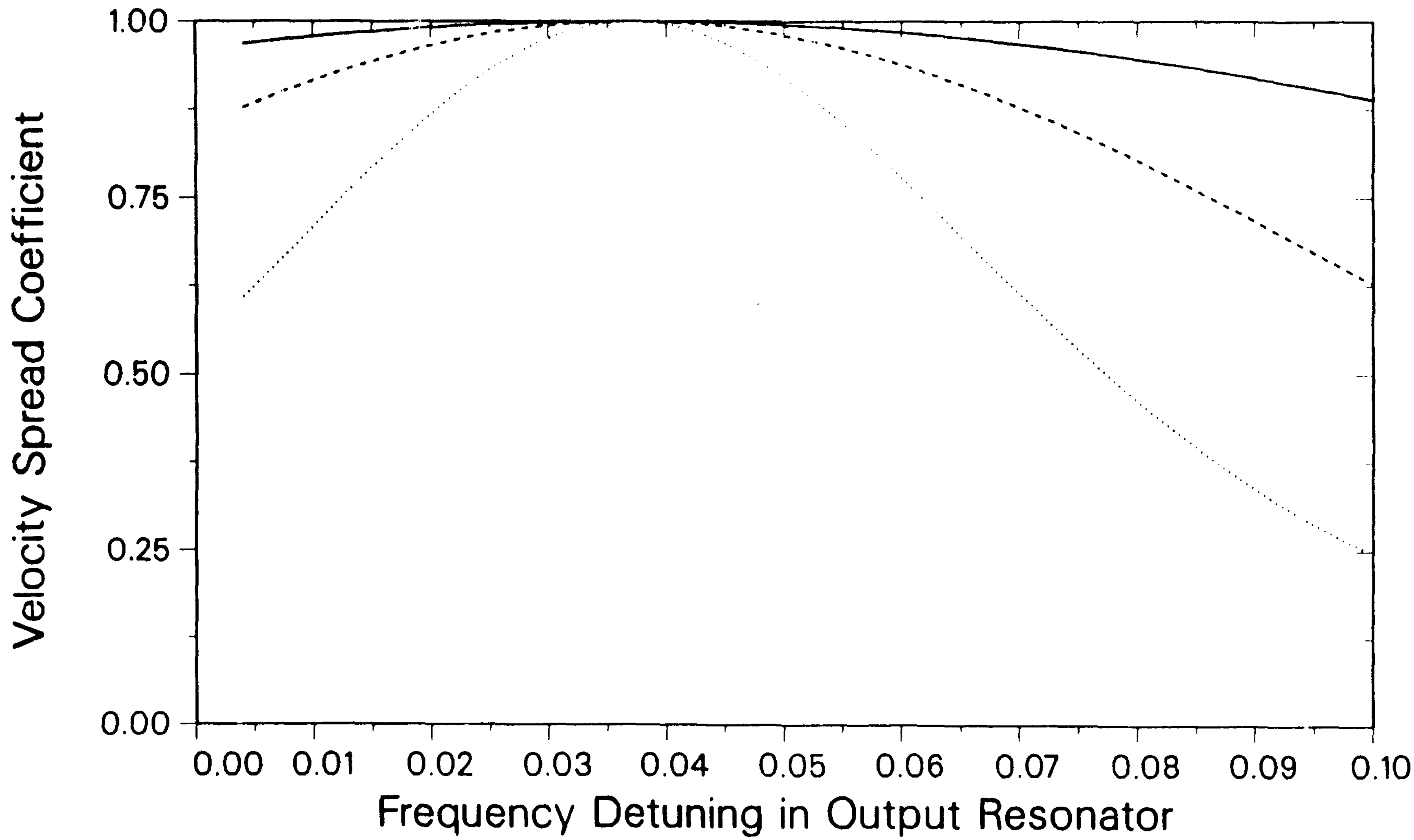

Fig. 6 — Same as Figure 5 for $\varkappa_0 = 0.8$